Proceedings of the 11th Brazilian Congress of Thermal Sciences and Engineering -- ENCIT 2006
Braz. Soc. of Mechanical Sciences and Engineering -- ABCM, Curitiba, Brazil,- Dec. 5-8, 2006

Paper CIT06-0648

THERMALLY DEVELOPING NON-NEWTONIAN LAMINAR FLOW IN ECCENTRIC ANNULAR DUCTS

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Abstract. *Thermally developing laminar flow in eccentric annular ducts involving non-Newtonian power-law fluids is analyzed by using the Generalized Integral Transform Technique (GITT) to solve the associated energy equation. The mathematical formulation is constructed based on the cylindrical coordinates system in such a way that the solid surfaces are described in terms of internal and external radii as functions of the angular coordinate, thus avoiding discontinuities in the boundary conditions. This thermal problem is here analyzed under the boundary condition of prescribed wall temperature. Numerical results for the temperature field and Nusselt numbers were produced for different values of the governing parameters, i.e., eccentricity and aspect ratio along the thermal entry region, which were critically compared with previously reported ones, in order to illustrate the usefulness of the employed integral transform approach with automatic error control.*

Keywords. *Eccentric annular ducts, Thermally developing flow, Power-law fluids, Integral transforms.*

1. Introduction

The study of thermally developing flow in eccentric annular ducts is important mainly due to its frequent appearance in several industrial applications, such as in heat exchange devices including the most common double-pipe configuration. In this type of equipment, due to imperfections and tolerances in the manufacturing steps, the eccentricity eventually caused, may or not be important. However, there are few typical applications where this effect is more pronounced and produced on purpose. Oil and gas drilling wells, polymer and plastic extrusion processes and nuclear reactors are some of the situations that reflect the importance of eccentricity in annular passages. In addition, when dealing with purely viscous non-Newtonian fluids, the essential heat and fluid flow analysis is much less available in the open literature, while commonly required in different industries, namely, chemical, food processing and pharmaceutical. There, the power-law model can adequately describe the rheology of a wide variety of fluids.

The literature survey brings up several works that dealt with thermally developing or fully developed flow in this geometric configuration, such as the pioneering works of Piercy *et al.* (1933), Stevenson (1949), Snyder and Goldstein (1965) and Jonsson and Sparrow (1965), which concentrated their analyses in the fluid flow, while Cheng and Hwang (1968), Trombetta (1971) and Suzuki *et al.* (1991) analyzed the heat transfer problem under different sets of boundary conditions. Details of such earlier works can be found in the compilation of Shah and London (1978). These studies have regained interest in the more recent works of Manglik and Fang (1995), Fang *et al.* (1999), Manglik and Fang (2002) and Escudier *et al.* (2002) in which the effects of eccentricity and duct rotation were investigated for the flow and heat transfer of non-Newtonian fluids. Escudier *et al.* (2002), in addition, recently offered an excellent literature review for flow and heat transfer in eccentric annular ducts involving Newtonian and non-Newtonian fluids.

With the advance of a structured hybrid analytical-numerical approach for convection-diffusion problems along the last two decades, named the Generalized Integral Transform Technique (GITT), Cotta (1993) & Cotta (1994), including the solution of elliptic diffusion-type problems defined within irregular domains (Aparecido *et al.*, 1989), it was possible to apply this analysis to fully developed laminar flow within ducts of various shapes, such as trapezoidal, triangular, and hexagonal ducts (Aparecido and Cotta, 1987; Aparecido *et al.*, 1989; Aparecido and Cotta, 1990; Barbuto and Cotta, 1997). Fully developed laminar flow and heat transfer of non-Newtonian fluids inside irregular ducts of different geometric configurations was also treated (Chaves *et al.*, 2001a; 2001b; 2004, Monteiro *et al.*, 2004), again through extension of the GITT approach, yielding accurate numerical results for quantities of practical interest such as the Fanning friction factor and Nusselt numbers, within a wide range of the governing parameters.

The present study is aimed at applying the so-called Generalized Integral Transform Technique (GITT) to solve the energy equation for thermally developing laminar flow of power-law fluids inside eccentric annular ducts subjected to constant temperatures either at the inner or outer duct walls. The cylindrical coordinates system is used in the mathematical formulation, so that the solid surfaces are described in the form of internal and external radii as functions of the angular coordinate, and thus avoiding more involved formulation in other coordinates systems and cumbersome domain transformation approaches. An analysis of convergence of the eigenfunction expansion is performed and a set of benchmark results is established for quantities of practical interest, such as dimensionless average temperature and local Nusselt numbers, within a wide range of the dimensionless axial coordinate, different power-law indices, aspect ratios and dimensionless eccentricities. Comparisons are then critically performed with previously reported results from direct numerical approaches along both, fully developed and thermally developing regions.

2. Analysis

Thermally developing flow of a non-Newtonian fluid inside eccentric annular ducts is considered according to Fig. (1). The non-Newtonian viscosity expression $\eta = K\gamma^{n-1}$ is described according to the Ostwald-de Waele model or power law model (Bird *et al.*, 1987), where K is the fluid consistency index (given in $N.s^n/m^2$), n is the power-law index (dimensionless) and γ is the rate-of-deformation tensor. The power-law fluid classification is (according to the n value): pseudoplastic ($n < 1$), Newtonian ($n = 1$ and $K = \mu$) and dilatant fluids ($n > 1$). This rheological model was employed to obtain the fully developed velocity profile in eccentric annular ducts in a previous work by Monteiro *et al.* (2004), which is used as input for the present thermal problem formulation. In addition, the duct walls are subjected to different situations of prescribed temperature. Therefore, the dimensionless energy equation for constant properties flow, neglecting axial conduction and viscous dissipation, is written as:

$$W(R,\theta)\frac{\partial T}{\partial Z} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2}, \quad \text{in} \quad Z > 0,\ R_1(\theta) < R < R_2(\theta),\ 0 < \theta < \pi \tag{1a}$$

with inlet and boundary conditions given, respectively, as follows:

$$T(R,\theta,0) = 0, \qquad R_1(\theta) \le R \le R_2(\theta), \qquad 0 \le \theta \le \pi \tag{1b}$$

$$\left.\begin{array}{l} T(R_1(\theta),\theta,Z) = 1, \quad \dfrac{\partial T(R_2(\theta),\theta,Z)}{\partial \mathbf{n}} = 0, \quad Z > 0, \quad \text{Case 1A} \\ \dfrac{\partial T(R_1(\theta),\theta,Z)}{\partial \mathbf{n}} = 0, \quad T(R_2(\theta),\theta,Z) = 1, \quad Z > 0, \quad \text{Case 1B} \end{array}\right\} \tag{1c-f}$$

$$\frac{\partial T(R,0,Z)}{\partial \theta} = 0, \quad \frac{\partial T(R,\pi,Z)}{\partial \theta} = 0, \quad Z > 0 \tag{1g,h}$$

where $\partial/\partial\mathbf{n}$ represents the normal derivative to the channel wall surface, in the sense leaving the medium.

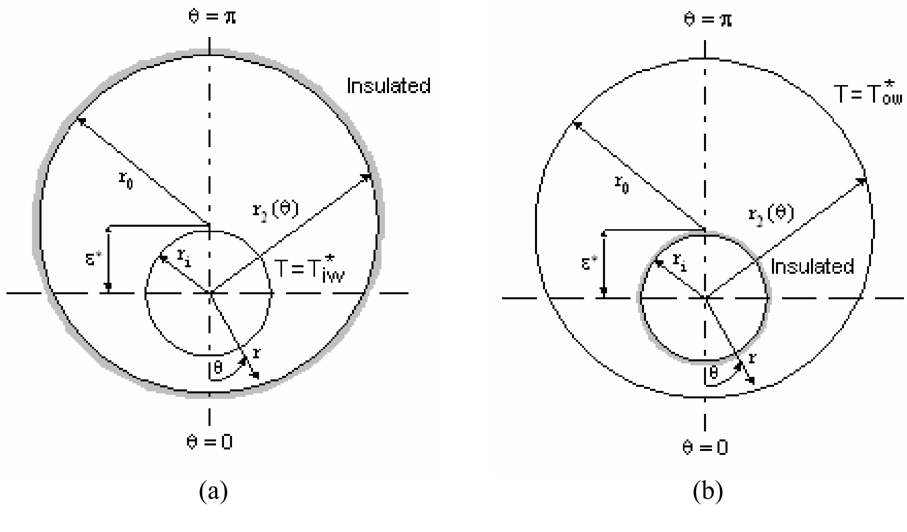

Figure 1 - Geometry and coordinates system for thermally developing flow in eccentric annular ducts: (a) prescribed temperature at the inner wall; (b) prescribed temperature at the outer wall.

The following dimensionless groups were employed in Eqs. (1) above:

$$R = \frac{r}{r_o},\ R_1(\theta) = \frac{r_1(\theta)}{r_o},\ R_2(\theta) = \frac{r_2(\theta)}{r_o} = \sqrt{1-\varepsilon^2(1-\gamma)^2\sin^2\theta} - \varepsilon(1-\gamma)\cos\theta,\ \gamma = \frac{r_i}{r_o},\ \varepsilon = \frac{\varepsilon^*}{r_o - r_i},\ Z = \frac{z}{D_h Pe} \quad \text{(2a-f)}$$

$$V_z(R,\theta) = \frac{v_z(r,\theta)}{\left[\left(-\frac{dP}{dZ}\right)\frac{D_h^{n+1}}{K}\right]^{1/n}},\ W(R,\theta) = \frac{V_z(R,\theta)}{4(1-\gamma)^2 V_{z,m}},\ Pe = Re\,Pr = \frac{v_{z,m}D_h}{\alpha} = \frac{\rho c_p}{k}v_{z,m}D_h \quad \text{(2h-i)}$$

$$Re = \frac{\rho v_{z,m}^{2-n}D_h^n}{K},\ Pr = \frac{K}{\rho v_{z,m}^{1-n}D_h^{n-1}\alpha},\quad \left.\begin{aligned} T(R,\theta,Z) &= \frac{T^*(r,\theta,z) - T_e}{T_{iw}^* - T_e}, \quad \text{Case 1A} \\ T(R,\theta,Z) &= \frac{T^*(r,\theta,z) - T_e}{T_{ow}^* - T_e}, \quad \text{Case 1B} \end{aligned}\right\} \quad \text{(2j-m)}$$

The main dimensionless groups in Eqs. (2) above are: $R_2(\theta)$ (dimensionless function that describes the outer surface), γ (aspect ratio), ε (dimensionless eccentricity), Pe (Péclet number), Re (Reynolds number) and Pr (Prandtl number). D_h is the hydraulic diameter defined as $D_h = 2r_o(1-\gamma)$.

In order to homogenize Eqs. (1c) and (1f), and thus improve the computational performance, a filter is defined as:

$$T(R,\theta,Z) = 1 + \Phi(R,\theta,Z) \quad \text{(3)}$$

After introducing Eq. (3) into Eqs. (1), the following problem for the potential $\Phi(R,\theta,Z)$ results:

$$W(R,\theta)\frac{\partial\Phi}{\partial Z} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\theta^2},\ \text{ in }\ Z>0,\ R_1(\theta) < R < R_2(\theta),\ 0<\theta<\pi \quad \text{(4a)}$$

$$\Phi(R,\theta,0) = -1,\qquad R_1(\theta) \le R \le R_2(\theta),\qquad 0\le\theta\le\pi \quad \text{(4b)}$$

$$\left.\begin{aligned} &\Phi(R_1(\theta),\theta,Z) = 0,\quad \frac{\partial\Phi(R_2(\theta),\theta,Z)}{\partial\mathbf{n}} = 0,\quad Z>0,\quad \text{Case 1A} \\ &\frac{\partial\Phi(R_1(\theta),\theta,Z)}{\partial\mathbf{n}} = 0,\quad \Phi(R_2(\theta),\theta,Z) = 0,\quad Z>0,\quad \text{Case 1B} \end{aligned}\right\} \quad \text{(4c-f)}$$

$$\frac{\partial\Phi(R,0,Z)}{\partial\theta} = 0,\quad \frac{\partial\Phi(R,\pi,Z)}{\partial\theta} = 0,\quad Z>0 \quad \text{(4g,h)}$$

The dimensionless velocity profile is given from the solution of the flow equations by the application of the GITT approach itself, for a non-Newtonian power-law fluid flowing within eccentric annular ducts, as an infinite series in the form (Monteiro *et al.*, 2004):

$$W(R,\theta) = \sum_{i=1}^{\infty}\Psi_i(R,\theta)\overline{V}_{Z,i}(\theta),\quad \overline{V}_{Z,i}(\theta) = \frac{1}{L_i(\theta)}\int_{R_1(\theta)}^{R_2(\theta)}\frac{\Psi_i(R,\theta)V_Z(R,\theta)}{R}dR \quad \text{(5a,b)}$$

$$\Psi_i(R,\theta) = \sin\left\{\lambda_i(\theta)\ln\left[\frac{R_2(\theta)}{R}\right]\right\},\quad \lambda_i(\theta) = \frac{i\pi}{\ln\left[\frac{R_2(\theta)}{R_1(\theta)}\right]},\quad L_i(\theta) = \frac{1}{2}\ln\left[\frac{R_2(\theta)}{R_1(\theta)}\right],\quad i=1,2,3,\ldots \quad \text{(5c-e)}$$

In Eqs. (5), the quantities $\overline{V}_{Z,i}(\theta)$ represent the transformed potentials for the velocity field, which were numerically obtained through appropriate subroutines such as DBVPFD from the IMSL Library (1991), with local error control.

Due to the non-separable nature of the velocity profile given in Eq. (4a) and consequently, of the related eigenvalue problem needed to solve the energy equation through well-known analytical methods such as the classical integral transform technique (Mikhailov and Ösizik, 1984), an exact solution of problem (4) is not possible. On the other hand, with the advances on the so-called GITT approach for the hybrid analytical-numerical solution of this class of non-transformable problem, it is possible to avoid these difficulties as now demonstrated (Aparecido and Cotta, 1992; Cotta, 1993, Chaves et al., 2002). For this purpose, in order to alleviate the difficulties related to the eigenvalue problem and to progress with the application of the generalized integral transform technique, the following auxiliary eigenvalue problems are chosen:

R-direction:

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\frac{\partial \Gamma_i(R,\theta)}{\partial R}\right]+\mu_i^2(\theta)\frac{1}{R^2}\Gamma_i(R,\theta)=0,\quad R_1(\theta)<R(\theta)<R_2(\theta) \tag{6a}$$

$$\left.\begin{array}{l} \Gamma_i\left[R_1(\theta),\theta\right]=0,\quad \dfrac{\partial\Gamma_i\left[R_2(\theta),\theta\right]}{\partial R}=0,\quad \text{Case 1A} \\ \dfrac{\partial\Gamma_i\left[R_1(\theta),\theta\right]}{\partial R}=0,\quad \Gamma_i\left[R_2(\theta),\theta\right]=0,\quad \text{Case 1B} \end{array}\right\} \tag{6b-e}$$

θ-direction:

$$\frac{d^2\varphi_\ell(\theta)}{d\theta^2}+\beta_\ell^2\varphi_\ell(\theta)=0,\quad 0<\theta<\pi \tag{7a}$$

$$\frac{d\varphi_\ell(0)}{d\theta}=0,\quad \frac{d\varphi_\ell(\pi)}{d\theta}=0 \tag{7b,c}$$

which are readily solved to yield eigenfunctions and eigenvalues, as follows:

$$\left.\begin{array}{l} \Gamma_i(R,\theta)=\cos\left\{\mu_i(\theta)\ln\left[\dfrac{R_2(\theta)}{R}\right]\right\},\quad \text{Case 1A} \\ \Gamma_i(R,\theta)=\sin\left\{\mu_i(\theta)\ln\left[\dfrac{R_2(\theta)}{R}\right]\right\},\quad \text{Case 1B} \end{array}\right\},\quad \varphi_\ell(\theta)=\cos(\beta_\ell\theta) \tag{8a-c}$$

$$\mu_i(\theta)=\frac{(2i-1)\pi}{2}\frac{1}{\ln\left[\dfrac{R_2(\theta)}{R_1(\theta)}\right]},\quad i=1,2,3...,\quad \beta_\ell=\ell-1,\quad \ell=1,2,3... \tag{8d,e}$$

Also, the eigenfunctions above enjoy the following orthogonality properties:

$$\int_{R_1(\theta)}^{R_2(\theta)}\frac{\Gamma_i(R,\theta)\Gamma_j(R,\theta)}{R}dR=\begin{cases}0, & i\neq j\\ M_i(\theta), & i=j\end{cases},\quad M_i(\theta)=\int_{R_1(\theta)}^{R_2(\theta)}\frac{\Gamma_i^2(R,\theta)}{R}dR=\frac{1}{2}\ln\left[\frac{R_2(\theta)}{R_1(\theta)}\right] \tag{9a,b}$$

$$\int_0^\pi\varphi_\ell(\theta)\varphi_m(\theta)d\theta=\begin{cases}0, & \ell\neq m\\ N_\ell, & \ell=m\end{cases},\quad N_\ell=\int_0^\pi\varphi_\ell^2(\theta)d\theta=\begin{cases}\pi, & \ell=1\\ \dfrac{\pi}{2}, & \ell>1\end{cases} \tag{9c,d}$$

Eigenvalue problems (6) and (7) allow the development of the following integral transform pair:

$$\tilde{\bar{\Phi}}_{i\ell}(Z)=\int_0^\pi\int_{R_1(\theta)}^{R_2(\theta)}\frac{\Gamma_i(R,\theta)\varphi_\ell(\theta)\Phi(R,\theta,Z)}{R\,M_i(\theta)N_\ell}dRd\theta,\qquad \text{transform} \tag{10a}$$

$$\Phi(R,\theta,Z)=\sum_{i=1}^\infty\sum_{\ell=1}^\infty\Gamma_i(R,\theta)\varphi_\ell(\theta)\tilde{\bar{\Phi}}_{i\ell}(Z),\qquad \text{inverse} \tag{10b}$$

The next step in the GITT approach is the integral transformation process itself, when all independent variables are eliminated from the partial differential formulation but one, in this case the dimensionless axial coordinate. To obtain the resulting system of ordinary differential equations for the transformed potentials $\tilde{\bar{\Phi}}_{i\ell}(Z)$, the partial differential equation (4a) is multiplied by $\Gamma_i(R,\theta)\varphi_\ell(\theta)/R$, integrated over the domains $[R_1(\theta),R_2(\theta)]$ in the R-direction and $[0,\pi]$ in the θ-direction, and the inverse formula, Eq. (10b), is employed in place of the filtered temperature distribution $\Phi(R,\theta,Z)$, resulting in the following transformed ordinary differential system:

$$\sum_{j=1}^\infty\sum_{m=1}^\infty F_{ij\ell m}\frac{d\tilde{\bar{\Phi}}_{jm}(Z)}{dZ}+\sum_{j=1}^\infty\sum_{m=1}^\infty G_{ij\ell m}\tilde{\bar{\Phi}}_{jm}(Z)=0,\quad Z>0\,;\; j=1,2,...\;\; m=1,2,... \tag{11a}$$

$$\tilde{\bar{\Phi}}_{i\ell}(0)=\tilde{\bar{h}}_{i\ell} \tag{11b}$$

where,

$$F_{ij\ell m} = \int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} \frac{R\Gamma_i(R,\theta)\Gamma_j(R,\theta)W(R,\theta)\varphi_\ell(\theta)\varphi_m(\theta)}{M_i(\theta)N_\ell} dRd\theta, \quad G_{ij\ell m} = \beta_\ell^2\delta_{ij}\delta_{\ell m} + G1_{ij\ell m} - G2_{ij\ell m} - 2G3_{ij\ell m} \tag{11c,d}$$

$$\tilde{\tilde{h}}_{i\ell} = -\int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} \frac{\Gamma_i(R,\theta)\varphi_\ell(\theta)}{RM_i(\theta)N_\ell} dRd\theta\,, \quad G1_{ij\ell m} = (p-1)\int_0^{\pi} \frac{\varphi_\ell(\theta)\varphi_m'(\theta)}{M_i(\theta)N_\ell} \frac{\varepsilon(1-\gamma)\sin\theta}{\sqrt{1-\varepsilon^2(1-\gamma)^2\sin^2\theta}} d\theta + \delta_{ij}\int_0^{\pi}\mu_i^2(\theta)\frac{\varphi_\ell(\theta)\varphi_m(\theta)}{N_\ell} d\theta \tag{11e,f}$$

$$G2_{ij\ell m} = \int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} \left[\frac{\Gamma_i(R,\theta)}{R}\frac{\partial^2\Gamma_j(R,\theta)}{\partial\theta^2}\frac{\varphi_\ell(\theta)\varphi_m(\theta)}{M_i(\theta)N_\ell}\right] dRd\theta\,, \quad G3_{ij\ell m} = \int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} \left[\frac{\Gamma_i(R,\theta)}{R}\frac{\partial\Gamma_j(R,\theta)}{\partial\theta}\frac{\varphi_\ell(\theta)\varphi_m'(\theta)}{M_i(\theta)N_\ell}\right] dRd\theta \tag{11g,h}$$

$$p = \begin{cases} 0, & \text{Case 1A} \\ 1, & \text{Case 1B} \end{cases} \tag{11i}$$

In Eq. (11a) each summation is associated with the eigenfunction expansion in a corresponding spatial coordinate, for computational purposes, the series solution given by Eq. (10b) is, in general, truncated to a finite number of terms in the summation, in order to compute the potential $\Phi(R,\theta,Z)$. The solution convergence is verified by comparing the values for the potential obtained with the truncated series for different numbers of retained terms. The coupled system of ordinary differential equations (11a) is solved by efficient numerical algorithms for initial value problems, such as in subroutine DIVPAG from the IMSL package (1991), with high accuracy and automatic control along the solution procedure. Then, after the transformed potentials are numerically obtained, quantities of practical interest are determined from the analytical inverse formula (10b), such as the dimensionless average temperature

$$T_{av}(Z) = 1 + \frac{2}{A_t(\gamma)}\int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} \frac{RV_Z(R,\theta)}{V_{Z,m}}\Phi(R,\theta,Z)dRd\theta \tag{12a}$$

or,

$$T_{av}(Z) = 1 + \frac{2}{A_t(\gamma)V_{Z,m}}\sum_{i=1}^{\infty}\sum_{\ell=1}^{\infty} I_{i\ell}\tilde{\bar{\Phi}}_{i\ell}(Z) \tag{12b}$$

where $A_t(\gamma)$ is the cross-section area of the annular passage and,

$$I_{i\ell} = \sum_{k=1}^{\infty}\int_0^{\pi}\int_{R_1(\theta)}^{R_2(\theta)} R\Gamma_i(R,\theta)\Psi_k(R,\theta)\varphi_\ell(\theta)\bar{V}_{Z,k}(\theta)dRd\theta \tag{12c}$$

The local Nusselt number can be calculated by making use of the temperature gradients at the walls integrated over the perimeter, or utilizing the axial gradient of the average temperature,

$$Nu_w(Z) = \frac{h(z)D_h}{k} = -\frac{2(1\text{-}\gamma)}{\pi}\frac{\int_0^{\pi}\left(\frac{\partial T}{\partial \mathbf{n}}\right)_{R_w(\theta)} d\theta}{1\text{-}T_{av}(Z)}\,, \quad w = \begin{cases} 1 & \text{to} \quad \text{Case 1A} \\ 2 & \text{to} \quad \text{Case 1B} \end{cases} \tag{13a,b}$$

or,

$$Nu_1(Z) = \frac{1}{4}\frac{(1+\gamma)}{\gamma}\frac{\frac{dT_{av}}{dZ}}{1\text{-}T_{av}} \quad \text{(Case 1A)} \qquad Nu_2(Z) = \frac{(1+\gamma)}{4}\frac{\frac{dT_{av}}{dZ}}{1\text{-}T_{av}} \quad \text{(Case 1B)} \tag{13c,d}$$

3. Results and discussion

Numerical results for thermally developing laminar flow of power-law fluids inside eccentric annular ducts were obtained by codes developed in the FORTRAN 90 programming language. The system given by Eqs. (11) was handled through the subroutine DIVPAG from the IMSL Library (1991) with a user-prescribed subroutine relative error target of 10^{-8}. These codes were implemented on a PENTIUM – IV 1.3 GHz microcomputer and the complete solution was computed with NT ≤ 400 in the expansions. The results are presented in terms of dimensionless average temperatures and local Nusselt numbers along the dimensionless axial coordinate, within the range $Z = 10^{-3}$ to 1, for different values of power-law indices, dimensionless eccentricity and aspect ratios.

Table (1) illustrates the convergence behavior of the present approach in terms of the local Nusselt number in the thermal entry region (i. e., $Z = 10^{-3}$, 10^{-1} and 1) for different power-law indices and dimensionless eccentricity, $\varepsilon = 0$. It is observed an excellent convergence behavior, with practically three converged digits for all positions considered.

Table 1. Convergence of the local Nusselt number for an eccentric annular duct with dimensionless eccentricity ε = 0.

γ	NT	Case 1A n = 0.5	Case 1A n = 1.0	Case 1A n = 1.5	Case 1B n = 0.5	Case 1B n = 1.0	Case 1B n = 1.5
Z = 0.001							
0.2	25	36.190	33.814	32.954	10.201	9.9638	9.8819
	100	43.855	34.798	32.183	17.433	17.077	16.946
	255	40.141	33.676	31.832	23.077	22.450	22.212
	400	40.371	33.715	31.792	26.489	25.416	25.009
0.5	25	23.043	22.344	22.035	13.365	13.073	12.969
	100	34.203	31.453	30.180	22.846	22.215	21.958
	255	32.835	29.008	27.519	28.115	26.762	26.196
	400	31.464	28.436	27.239	29.091	27.010	26.171
0.8	25	18.830	18.346	18.149	15.794	15.429	15.291
	100	30.150	28.544	27.825	26.404	25.417	24.995
	255	31.800	27.837	27.460	30.357	28.206	27.305
	400	29.931	27.051	25.973	29.532	26.854	25.825
Z = 0.1							
0.2	25	10.714	10.022	9.8155	5.7154	5.5408	5.4891
	100	10.610	9.9157	9.7023	5.7034	5.4887	5.4225
	255	10.625	9.9290	9.7149	5.6843	5.4736	5.4088
	400	10.619	9.9244	9.7107	5.6798	5.4702	5.4058
0.5	25	7.5796	7.2741	7.1247	6.0831	5.8550	5.7676
	100	7.5289	7.2443	7.1025	5.9938	5.7682	5.6832
	255	7.5334	7.2479	7.1055	5.9876	5.7638	5.6794
	400	7.5311	7.2462	7.1041	5.9856	5.7623	5.6781
0.8	25	6.8205	6.5377	6.4088	6.3728	6.1141	6.0043
	100	6.7671	6.5101	6.3928	6.3005	6.0623	5.9627
	255	6.7678	6.5105	6.3935	4.7106	6.0607	5.9615
	400	6.7661	6.5097	6.3924	6.2964	6.0596	5.9605
Z = 1							
0.2	25	8.3698	8.1313	8.0802	4.2288	4.1945	4.2089
	100	8.3681	8.1297	8.0788	4.2287	4.1944	4.2087
	255	8.3681	8.1296	8.0787	4.2287	4.1944	4.2087
	400	8.3681	8.1296	8.0783	4.2287	4.1944	4.2087
0.5	25	5.7702	5.7385	5.7101	4.4536	4.4296	4.4292
	100	5.7696	5.7381	5.7099	4.2562	4.4293	4.4293
	255	5.7696	5.7381	5.7106	4.4535	4.4293	4.4293
	400	5.7696	5.7381	5.7100	4.4534	4.4293	4.4293
0.8	25	5.1098	5.0822	5.0641	4.7098	4.6850	4.6760
	100	5.1095	5.0821	5.0640	4.7097	4.6848	4.6760
	255	5.1095	5.0820	5.0640	4.7090	4.6850	4.6760
	400	5.1095	5.0820	5.0640	4.7097	4.6850	4.6760

Table (2) shows a comparison of the asymptotic Nusselt numbers for the case 1A against those of Manglik and Fang (2002). It can be noticed an excellent agreement among the results furnishing a direct validation of the computational code developed in the present work.

Table 2. Comparison of the asymptotic Nusselt number for an eccentric annular duct for the case 1A.

γ	ε	n = 0.5 Present	n = 0.5 Manglik and Fang (2002)	n = 1 Present	n = 1 Manglik and Fang (2002)	n = 1.5 Present	n = 1.5 Manglik and Fang (2002)
0.2	0	8.3681	8.1093	8.1296	8.0651	8.0783	8.0151
	0.2	6.5178	6.5244	6.4796	6.4861	6.4509	6.4574
	0.6	3.9721	3.9761	3.9875	3.9915	3.9812	4.0012
0.5	0	5.7696	5.7621	5.7385	5.7316	5.7100	5.7040
	0.2	3.5630	3.5666	3.7349	3.7386	3.8073	3.8264
	0.6	2.0318	2.0338	2.1018	2.1039	2.1396	2.1504
0.8	0	5.1095	5.1051	5.0820	5.0785	5.0640	5.0604
	0.2	2.5675	2.5701	2.8312	2.8369	2.9296	2.9443
	0.6	1.2897	1.2923	1.4173	1.4216	1.4725	1.4874

Figure (2) presents the evolution of the dimensionless average temperature along the thermal entry region for a Newtonian fluid with a dimensionless eccentricity ε = 0 and different aspect ratios. One can see that the aspect ratio causes an opposite behavior on this evolution; while for the case 1A the fully developed region is reached more rapidly for higher aspect ratios, for the case 1B this region is reached faster for lower values of this parameter. This can be explained by the effect of the adopted thermal boundary conditions and to a decrease in the annular gap; for the case 1A the insulated wall is the outer surface, so that the inner surface is subjected to higher heat fluxes but at lower aspect ratios, the average velocity is also lower, this way the heat exchange is less intensified and it is increased as the aspect ratio increases; while the opposite is observed for the case 1B.

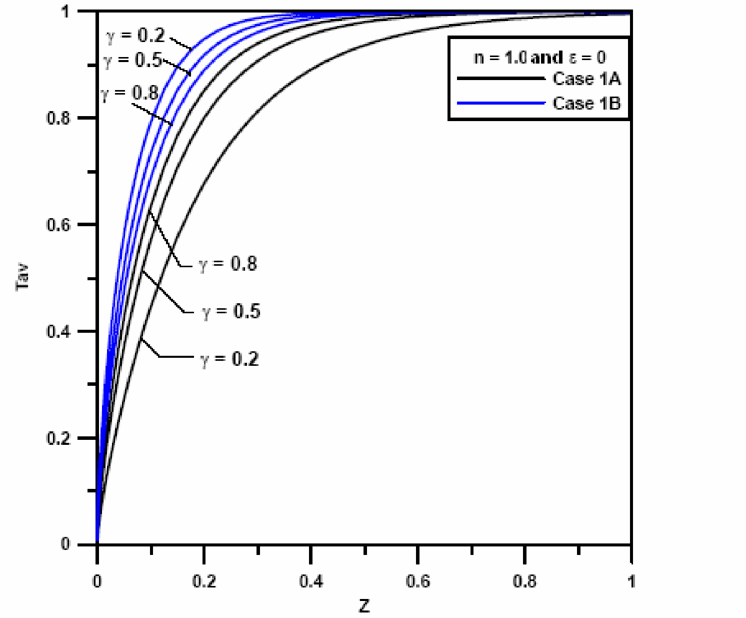

Figure 2. Evolution of the dimensionless average temperature along the thermal entry region.

Figure (3) presents the effect of the power-law index on the temperature field as a function of the normalized radial coordinate, for the two cases of thermal boundary conditions analyzed at the angular positions θ = 0° and 180°, and considering fixed values of axial position, aspect ratio and dimensionless eccentricity (Z = 1, γ = 0.2 and ε = 0.6). It is verified that for the case 1A the power-law index does not affect the temperature field for both angular positions. This can be explained by the fact of satisfying the energy equation leading to a compensation evidenced by flatten temperature distributions, once the heat transfer process is limited by the nearest adiabatic wall. For the second case (case 1B), the power-law index slightly affects the temperature profile, for lower values of the normalized axial coordinate at θ = 180°, due to an increase of the surface area near the adiabatic wall.

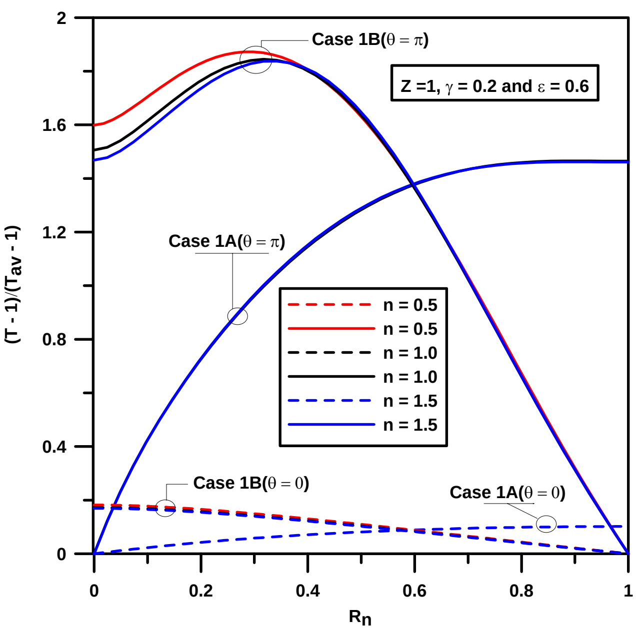

Figure 3. Effect of the power-law index on the temperature field for an eccentric annular duct.

Similarly, Fig. (4) brings the effect of the dimensionless eccentricity on the temperature field as a function of the normalized radial coordinate. It is noted that an increase of this parameter results in an increase of the stagnation zone; as a result it higher temperature gradients are experienced in this region. For the case where the adiabatic wall is far from this zone, the temperature gradient is lower, this way resulting in higher peaks of temperature for lower dimensionless eccentricities.

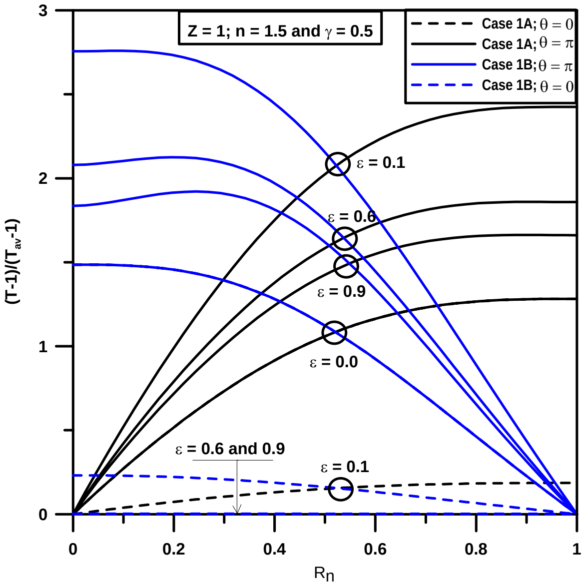

Figure 4. Effect of the dimensionless eccentricity on the temperature field for an eccentric annular duct.

Finally, Fig. (5) now considers the effect of the aspect ratio. It is evidenced that an increase of the aspect ratio results in higher peaks of temperature, as a direct consequence of a decrease on the annular gap width, so that higher temperature gradients are experienced mainly at the stagnation zones.

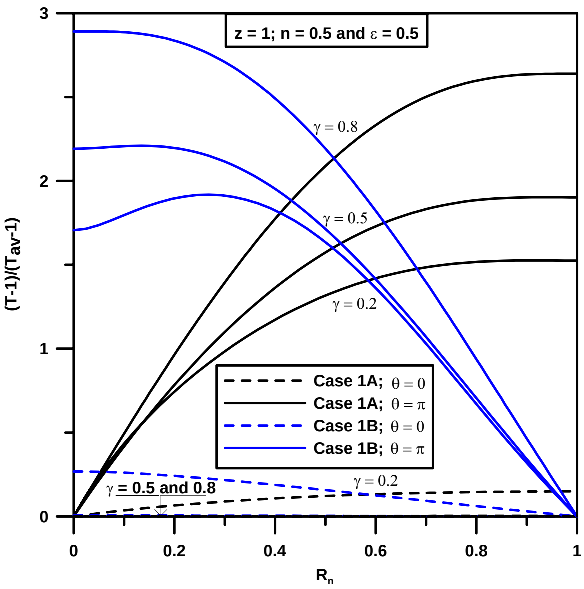

Figure 5. Effect of the aspect ratio on the temperature field for an eccentric annular duct.

4. Conclusions

The present study was intended to further demonstrate the usefulness of the Generalized Integral Transform Technique (GITT) as a benchmarking and covalidation tool in the simulation of convection-diffusion problems, while the codes created with this approach were found to be relatively cost-effective, within the range of truncations orders considered. Numerical results for convection heat transfer in eccentric annular ducts were tabulated and graphically presented providing a reliable source of benchmark for the local Nusselt numbers and dimensionless average temperature.

5. Acknowledgement

The authors wish to acknowledge the financial support provided by CNPq and FAPERJ.

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